

Comovement of Home Prices: A Conditional Copula Approach¹

Lei Hou, Wei Long, and Qi Li

Abstract Even though housing markets in different areas are relatively localized, regional home prices have become closely correlated and tend to be simultaneously affected by many national economic factors. In this paper, through the dynamic copula model, we confirm that regional home price dependence is time-varying and the conventional time-invariant copulas underestimate the degree of dependence during economic expansions and recessions. In essence, the U.S. residential real estate market has become more integrated since the mid-1980s. Using the conditional copula model, we further identify how the dependence among regional housing markets evolves along with some fundamental economic factors such as unemployment rate and interest rate. These findings can help investors and home buyers to better identify and evaluate the systematic risk in the nationwide housing market.

Key Words Comovement; Copula; Dependence; Home price.

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1. INTRODUCTION

The U.S. housing market over the past two decades fluctuated substantially and provides a vivid example about the formation and burst of an economic bubble. As pointed by Shiller (2007), the recent housing boom was led by the more than 10% yearly increase in several west coastal cities (Los Angeles, San Diego, San Francisco and Seattle), then spilled into other areas such as Boston and Denver and eventually became an unprecedented national boom. After 2006, home prices were rapidly depreciating and became stabilized until recently.

It is intuitive to understand the comovement of localized home prices within the same region. There is a sizable literature in finance about contagion and herding effect to explain such a phenomenon. For example, Allen and Gale (2000) find that investors tend to engage in risk shifting and this kind of behavior can lead to bubbles in asset prices and increase the probability of general default. Transactions in housing market are usually debt financed, therefore the probability of bubble formation and collapse is also relatively higher when a small liquidity preference shock hits the market. However, the recent boom in housing price is a nationwide phenomenon and the effect of state- or regional-specific factors should be relatively small. Del Negro and Otrok (2007) investigate how local components affect housing price and find that only historical comovement can be attributed to local components, while the expansionary monetary policy drive the recent housing boom. Besides monetary policy, the fluctuation in home prices also provides some insights about how housing markets from different geographic areas became correlated through several national economic factors. Fu (2007) explores the possible factors influencing the national home price movements and concludes that those factors, such as monetary policy, demographics, real economic activity and inflation rate, account for about one-fourth of local home price volatility. Kallberg, Liu and Pasquariello (2014) investigate the comovement among home prices for 14 metropolitan areas during 1992-2008 and confirm that the substantial home price increase in the sample period can be attributed to the covariation of several fundamental and systematic real and financial factors. Landier, Sraer and Thesmar (2017) show that the integration of banking industry explains up to one fourth of the rise in house price correlation.

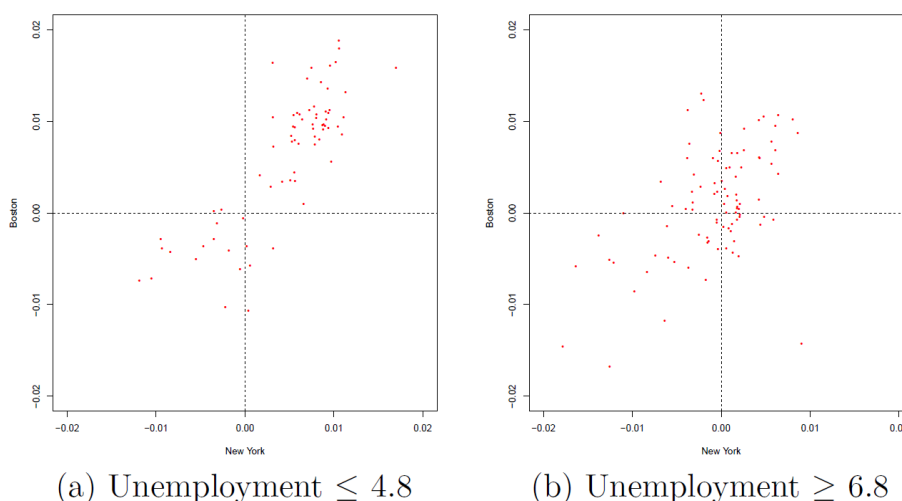
The main contribution of this study is that we implement the copula model to examine the dynamic dependence structure among different U.S. regions. It is the stylized fact that the correlation among financial markets is dynamic since the seminal work by Erb, Harvey and Viskanta

(1994) and Longin and Solnik (1995). Engle (2002) proposes a widely used dynamic correlation coefficient (DCC) method to detect and estimate the time-varying correlation coefficient between two financial assets. Kallberg et al. (2014) use a dynamic factor model to estimate how correlation coefficients among 14 largest cities in US evolve over time. Copula is another useful tool to estimate dependence structure. Copula theory is built on the Sklar's theorem (1959), which claims that a multivariate distribution can be fully characterized by its marginal distributions and a copula - a multivariate distribution function with uniform $[0,1]$ marginals. Compared with other methods such as DCC and Kallberg et al. (2014)'s dynamic factor model, copula has several superiorities. First, marginals of a copula could be from different distribution families and estimated nonparametrically, while DCC usually assumes a multivariate normal distribution or Student- t distribution and the dynamic factor model imposes some restrictions on residuals that cannot be explained by several common factors. Second, copula with high dimension can model the dependence among multiple assets simultaneously. DCC is built on multivariate GARCH models, but only pairwise dynamic correlation can be meaningfully obtained from the estimated correlation matrix. Third, different types of copula can estimate different dependence structure. For example, when a recession occurs and all financial markets crash simultaneously, a Clayton copula can be used to estimate the degree of dependence among these markets. Fourth, and of the greatest importance in the study of financial dependence, copula uses measures of concordance such as Kendall's τ and can capture the asymptotic tail dependence which is usually nonlinear, while both DCC and Kallberg et al. (2014)'s dynamic factor model only gives linear correlation coefficients, which tends to underestimate the degree of dependence when extreme events such as price boom or burst of bubble occurs. Since the recent fluctuation in housing price is extreme and substantial, copula model can be a useful tool to analyze the shift in dependence structures.

There is a growing literature about the application of copula model in economic research. Since Li (2000) and Embrechts et al. (2002), copula has been extensively used in finance and risk management. Besides, copula is also used to evaluate the comovement in housing prices. Zimmer (2012) finds that the widely-used Gaussian copula underestimates the interdependence across four heavily shocked housing markets (California, Nevada, Arizona and Florida) in the midst of the housing crisis, because the Gaussian copula predicts asymptotically independence for both tails and fails to capture the dependence across different areas when an extreme event happens (e.g., the

collapse of a housing bubble). He recommends a combination of the Gumbel and Clayton copula which has the ability to capture the dependence at both tails. Zimmer (2015) extends to higher dimensional copulas by using the vine copulas and concludes that multivariate vine copulas are more suitable to model comovement in home prices, but does not examine how the dependence changes along the business cycle. Figure 1 provides a direct but intuitive example about how the dependence structure of home prices in New York and Boston changes as the unemployment rate - arguably one of the most important economic indicators and closely tracked by investors and policy makers - evolves along the business cycle. Using the monthly Case-Shiller Index between 1990 and 2016 as the proxy of home prices in the two cities, in Figure 1a, we find that home prices tend to increase simultaneously when the unemployment rate is relatively low ($< 4.8\%$). On the contrary, Figure 1b indicates that housing markets in both cities tend to contract when lay-off becomes pervasive ($> 6.8\%$), because simultaneous decrease in home prices appears to be more frequent when the unemployment rate is high. This example implies the necessity of a conditional copula which can detect how the magnitude of dependence (copula's parameter) changes along with other covariates.

FIG. 1. Scatterplots of Case-Shiller Index in New York and Boston at different unemployment levels between 1990 and 2016.



In this study, we contribute to the literature by examining the dynamic dependence in housing markets across 9 U.S. census divisions through the conditional copula model, which is firstly proposed by Patton (2006) to estimate the dependence in exchange rate market and further extended by Hafner and Reznikova (2010), Acar, Craiu and Yao (2011) and Abegaz, Gijbels and Veraverbeke (2012). Under conditional copula, the dependence structure, measured by the copula parameter, is

allowed to be adjusted for covariates such as time and other economic variables. Since the real estate industry is an important component of the national account and home price is sensitive to other economic factors, conditional copula provides great flexibility for researchers to analyze how dependence structure evolves along with those economic factors. First, if the dependence indeed exists among regional housing markets, we would like to examine the pattern of the dependence path and how the degree of dependence changes over the past few decades, especially during booming and crisis periods when the linear correlation coefficient tends to underestimate the magnitude of dependence. Second, U.S. economy has experienced five economic recessions since the mid-1970s and macroeconomic factors such as GDP growth, unemployment rate and monetary policies have substantially changed. If the dependence structure across housing markets indeed correlates to the business cycle, we are interested in investigating how the degree of dependence evolves along with the level of those fundamental economic factors. To address the two questions, we respectively adopt the semiparametric dynamic copula model proposed by Hafner and Reznikova (2010) and the semiparametric estimation of conditional copula model by Abegaz, Gijbels and Veraverbeke (2012). Specifically, if $\tau \in (-1,1)$ denotes the correlation coefficient of housing markets in different areas, we aim to respectively extend it to $\tau(t)$ and $\tau(x)$: the former is a function of time t while the latter is a function of a covariate x .

We collect the 1975-2016 House Price Index for 9 U.S. census divisions from the Federal Housing Finance Agency. Our analysis confirms that home price dependence among the nine regions is dynamic: The comovement among regional housing markets was weak before the mid-1980s but became substantially strengthened since then. In other words, the U.S. residential real estate market has become more integrated since the mid- 1980s. The housing crisis in 2006 further consolidated the dependence structure and the association among regional housing market in 2016 was the highest since the end of 1990s. On the contrary, both DCC and Kallberg et al. (2014) underestimate the increased dependence during 1990s. Through conditional copula model, we also identify the relationship between the dependence structure and several fundamental economic factors. We find that the degree of dependence is stronger when the per capita personal real income decreases (increases) and when the unemployment is high (low). This is intuitive because economic crisis will dampen the demands for new houses while a boom will spur the demands. Loose monetary policy such as low interest rate will also spur demands for houses. We further find that comovement among

markets is evident when the ratio of debt payments in disposable income is high, which usually happens in times of boom or when the nationwide housing market is over-heated, like the bubbling period before the 2008 global economic crisis. However, when a crisis is looming and banks enforce stringent and less flexible loan policy, the deleveraging process will squeeze the percentage of residential investment in GDP and we find that regional housing markets will also exhibit strong dependence. Even though we cannot claim the causal relationship between the dependence and those economic factors because there exists many other local idiosyncratic factors which may also contribute to the dependence structure, the evolving patterns of dependence along with those factors can help investors and home buyers to analyze and identify the potential systematic risk before buying new houses.

The remaining parts of the paper are organized as follows. We briefly introduce basics of a copula model in Section 2. Section 3 discusses the semiparametric estimation of conditional copulas. Data and preliminary results for dynamic copula are included in Section 4 and Section 5. We estimate how dependence evolves along with a series of economic factors in Section 6. Section 7 concludes.

2. A BRIEF OF COPULA

Suppose we have a series of p -dimensional vectors of random variables $\{X_t\}_{t=1}^T$, where $X_t = (X_{t1}, \dots, X_{tp})'$. Let $F(x)$ and $f(x)$ be the joint distribution and density function of $X \in \mathbb{R}^p$, and $F_i(x_i)$ and $f_i(x_i)$ be the marginal distribution and density function of X_i , respectively, where $1 \leq i \leq p$. Then, $x_i = F_i^{-1}(u_i)$ where $F_i^{-1}(\cdot)$ is the inverse probability transformation function or quantile function for x_i and $u_i = F_i(x_i)$ is uniformly distributed over $[0,1]$ by the probability integral transformation. By Sklar (1959), the joint distribution of the p -dimensional vectors can be written as $F(x_1, x_2, \dots, x_p) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_p^{-1}(u_p)) = C(u_1, u_2, \dots, u_p)$ where $C(\cdot)$ is the copula function associated with the joint distribution. Thus, a copula is a multivariate distribution function with uniform marginal distributions on $[0,1]$. Sklar's theorem proves that, if F_1, F_2, \dots, F_p are continuous, the copula function defined above is unique. Conversely, for any marginal distribution F_i and any copula function C , the function $C(F_1(\cdot), F_2(\cdot), \dots, F_p(\cdot))$ is a multivariate distribution function with marginal distributions F_1, F_2, \dots, F_p . In other word, the

copula function C and all the marginal functions F_1, F_2, \dots, F_p are not necessarily of the same distribution family, which provides great flexibility when empirical users need to specify a multivariate distribution. In addition, according to Sklar's theorem, if C is continuous, one can separate the univariate marginals from the copula which represents the dependence structure. Specifically, if we assume $F_i(\bullet)$ is differentiable and C is p times differentiable, we have

$$\frac{\partial^p F(x_1, x_2, \dots, x_p)}{\partial x_1 \partial x_2 \dots \partial x_p} = c(u_1, u_2, \dots, u_p) \times f_1(x_1) \times \dots \times f_p(x_p)$$

where $c(u_1, u_2, \dots, u_p)$ is the density of $C(u_1, u_2, \dots, u_p)$. Thus, the density of F could be expressed as the product of the copula density and the univariate marginal densities and it is obvious that the copula has all the information about the dependence structure among the p dimensional vectors. A copula function is related to the joint cumulative distribution function via $C(F_1(x_1), F_2(x_2), \dots, F_p(x_p); \theta) = F(x_1, x_2, \dots, x_p)$, where the parameter θ characterizes dependence among the p covariates. This dependence parameter is closely related to the dependence measures such as Kendall's τ , Spearman's ρ and tail dependence coefficients. For application to financial data, the dependent parameter is of great interests as it describes the comovement of stocks. The estimation of the copula model is well studied, see Fan and Patton (2014) for a review of the copular model and their applications in economics.

There are many different types of copulas and they exhibit different dependence structure. In Table 1 we provide a summary of four widely-used copulas in empirical studies: Gaussian, Clayton, Gumbel and Frank. For the ease of exposition, Table 1 only displays bivariate copula cases and Nelsen (2006) provides more thorough and extensive summary of copulas. Gaussian copula is flexible in that it captures both positive and negative dependence. Clayton copula exhibits asymmetric dependence and is able to capture the lower tail dependence. Contrary to Clayton, Gumbel copula exhibits upper-tail dependence. For example, in the context of home price fluctuation, if we believe the housing markets tend to crash together during the recession period, the Clayton copula should be a better choice as it is able to exhibit the lower-tail dependence. For empirical users, one important question is how to choose a copula function that accurately specify the dependence structure among the marginals. For the goodness-of-fit tests and model selection, the widely used methods are the Kolmogorov-Smirnov (KS) test, the Cramér-von Mises (CvM) test

(see Rémillard, 2010) and the Bayesian Information Criterion (BIC) method. Since θ is not directly comparable among copulas, it is usually converted to measures of concordance such as Kendall's τ (Nelsen, 2006) which is bounded on $(-1, 1)$. Table 1 also documents how to convert θ to Kendall's τ for the four copulas.

TABLE 1.

Summary of Gaussian, Clayton, Gumbel and Frank Copula

Copula	Function Form	θ Domain	Kendall's τ
Gaussian	$C_{Gaussian}(u_1, u_2; \theta) = \Phi_G(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta)$	$-1 < \theta < 1$	$\frac{2}{\pi} \arcsin(\theta)$
Clayton	$C_{Clayton}(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$0 < \theta < \infty$	$\frac{\theta}{\theta+2}$
Gumbel	$C_{Gumbel}(u_1, u_2; \theta) = \exp(-(\tilde{u}_1^\theta + \tilde{u}_2^\theta)^{1/\theta})$	$1 \leq \theta < \infty$	$1 - \frac{1}{\theta}$
Frank	$C_{Frank}(u_1, u_2; \theta) = -\frac{1}{\theta} \log \left[1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1} \right]$	$\theta \in \mathbb{R} \setminus \{0\}$	$1 - \frac{4}{\theta} + \frac{4}{\theta^2} \int_0^\theta \frac{t}{e^t - 1} dt$

Note: Φ is the CDF of the standard normal and $\Phi_G(\cdot, \cdot)$ is the standard bivariate normal with correlation parameter θ . $\tilde{u}_1 = -\log u_1$ and $\tilde{u}_2 = -\log u_2$.

3. SEMIPARAMETRIC ESTIMATION TO CONDITIONAL COPULAS

A conditional copula becomes necessary when there exists a covariate Z whose influence on the dependence structure among X_1, X_2, \dots, X_p can be modelled by $C(u_1, u_2, \dots, u_p; \theta(z))$, where $\theta(z)$ is a function of the covariate. Put differently, if such a covariate exists, the dependence of C on z is fully determined by the dependence of θ on z . Therefore, our main target is to estimate the unknown function $\theta(z)$ which can be modelled as a polynomial of degree q . As mentioned in the previous section, the parameter space Θ is different for many copula families, while polynomial function can take any values on the real line. Acar, Craiu and Yao (2011) and Abegaz, Gijbels and Veraverbeke (2012) respectively propose a semiparametric method to estimate conditional copulas. Both suggest to use a transformation function $\psi\{\theta(z)\} = \eta(z)$ such that $\theta(z) = \psi^{-1}\{\eta(z)\} \in \Theta$ if ψ^{-1} exists. Manner and Reznikova (2012) provide a list of transformation functions for each copula family. Assuming that the $(q + 1)$ derivative of the function η exists for each point z , for an observation Z_i in the neighborhood of z , following Acar et al. (2011) and Abegaz et al. (2012), we approximate $\eta(Z_i)$ by a Taylor expansion:

$$\eta(Z_i) \approx \eta(z) + \eta'(z)(Z_i - z) + \dots + \eta^{(q)}(z)(Z_i - z)^q / q! \equiv \beta_0 + \beta_1(Z_i - z) + \dots + \beta_q(Z_i - z)^q$$

where $\beta_j = \eta^{(j)}(z) / j!$ for $j = 0, 1, 2, \dots, q$. We further define the joint and continuous marginal distribution of (X_1, X_2, \dots, X_p) , conditionally on $Z = z$, as

$$F_z(X_1, X_2, \dots, X_p) = P(X_1 \leq x_1, X_2 \leq x_2, \dots, X_p \leq x_p | Z = z),$$

$$F_{1z}(X_1) = P(X_1 \leq x_1 | Z = z), \dots, F_{pz}(X_p) = P(X_p \leq x_p | Z = z)$$

Then, if Z_t is near z , $c\{F_{1z}(X_{1t}), F_{2z}(X_{2t}), \dots, F_{pz}(X_{pt}) | \theta(Z_t)\}$ can be approximated as

$$c\{\hat{F}_{1z}(X_{1t}), \hat{F}_{2z}(X_{2t}), \dots, \hat{F}_{pz}(X_{pt}) | \psi^{-1}(\eta(Z_t))\} = c\{\hat{F}_{1z}(X_{1t}), \hat{F}_{2z}(X_{2t}), \dots, \hat{F}_{pz}(X_{pt}) | \psi^{-1}(\beta_0 + \beta_1(Z_t - z) + \dots + \beta_q(Z_t - z)^q)\}$$

where \hat{F}_{jz} denotes the estimated conditional distribution of X_j given $Z = z$ for $j = 1, 2, \dots, p$. In

Acar et al. (2011), the marginals are assumed to be known, which is very unlikely in empirical applications. Abegaz et al. (2012) extend to estimate the unknown marginals by the Nadaraya-

Watson estimator. Specifically, $\hat{F}_{jz}(x) = \sum_{t=1}^T \omega_t(z) \cdot I(X_{jt} \leq x)$, where $\sum_{t=1}^T \omega_t = 1$ or holds

$$\text{asymptotically, } \omega_t(z) = \frac{K_h(Z_t - z)}{\sum_{t=1}^T K_h(Z_t - z)}, \quad K_h(\cdot) = K(\cdot/h)/h \text{ is a kernel function with}$$

bandwidth h and $I(\cdot)$ is an indicator function.

After estimating the nonparametric estimates of the marginals, Abegaz et al. (2012) propose to maximize the following copula-based local pseudo log-likelihood function:

$$L(\beta) = \sum_{t=1}^T \log c\left[\hat{F}_{1z}(X_{1t}), \dots, \hat{F}_{pz}(X_{pt}) | \psi^{-1}(\beta_0 + \beta_1(Z_t - z) + \dots + \beta_q(Z_t - z)^q)\right] K_{h_T}(Z_t - z) \quad (1)$$

with respect to $\beta = (\beta_0, \beta_1, \dots, \beta_q)'$. The smoothing parameter h in equation (1) depends on the

sample size T : the sequence $h = h_T$ converges to zero as T extends to infinity. Denoting the local

polynomial maximum pseudo log-likelihood estimators as $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_q)'$, we can estimate

$\eta(z)$ and its derivatives $\hat{\eta}^{(j)}(z)$ through $\eta^j(z) = j! \hat{\beta}_j$ for $j = 0, 1, 2, \dots, q$. Consequently,

the estimator of $\theta(z)$ can be written as $\hat{\theta}(z) = \psi^{-1}(\hat{\eta}(z)) = \psi^{-1}(\beta_0)$ and then an estimator

of copula function $C(u_1, \dots, u_p | \theta(z))$ at data point z can be obtained. Abegaz et al. (2012)

suggest a sufficiently fine grid of z -values in the definition domain of the covariate Z to estimate the

entire function $\theta(z)$. Under some standard conditions, Abegaz et al. (2012) prove that, as $T \rightarrow \infty$,

there exists solutions $\hat{\beta}$ of the log-likelihood equations $\partial L(\beta) / \partial \beta_j = 0$ for all $j = 0, 1, 2, \dots,$

q such that $\hat{\beta}_j$ is consistent for estimating $\eta_j(z) = \eta^{(j)}(z) / j!$ for $j = 0, 1, \dots, q$.

In practice, there are two important issues on using semiparametric estimation methods. The first

is the bandwidth selection. Abegaz et al. (2012) derive the theoretically optimal bandwidth and provide a rule-of-thumb practical bandwidth selector.² Besides, they also mention two alternative classic methods: leave-one-out cross validation (LOOCV) and Akaike Information Criterion (AIC). In our analysis we mainly use the LOOCV to obtain the optimal bandwidth. The second issue is model selection. The copula family is numerous and it is crucial for empirical users to know which copula is the appropriate one. But in the real world, the true copula is unknown to researchers and the accuracy of the estimation is not directly comparable among different copulas. Therefore, we need to set a criterion for the model selection problem. In this study, we use the cross-validated prediction error (CVPE) method proposed in Acar et. al (2011). Suppose we want to choose the best one from the candidate copula set $\{C_k : k = 1, 2, \dots, K\}$. For the k^{th} candidate, the selected optimal bandwidth is denoted to be h_{opt}^k and the estimated conditional copula's parameter is $\hat{\theta}_{h_{opt}^k}^{(-t)}$ with the point $(X_{1t}, X_{2t}, \dots, X_{pt}, Z_t)$ left out. Correspondingly, the k^{th} candidate copula model can be written as $C_k(u_{1t}, u_{2t}, \dots, u_{pt} | \hat{\theta}_{h_{opt}^k}^{(-t)}(Z_t))$ with $t = 1, 2, \dots, T$ and $k = 1, 2, \dots, K$. For the joint distribution of u_1, u_2, \dots, u_p , we use the conditional expectation to estimate the predictive ability for each copula in the candidate set. Specifically, for u_1 ,

$$\hat{E}_k^{(-t)}(u_{1t} | u_{2t}, \dots, u_{pt}, Z_t) = \int_0^1 u_1 c_k(u_1, u_{2t}, \dots, u_{pt} | \hat{\theta}_{h_{opt}^k}^{(-t)}(Z_t)) du_1$$

Thus, the CVPE or the model selection criterion is defined as

$$CVPE(C_k) = \sum_{t=1}^T \left\{ \left[u_{1t} - \hat{E}_k^{(-t)}(u_{1t} | u_{2t}, \dots, u_{pt}, Z_t) \right]^2 + \dots + \left[u_{pt} - \hat{E}_k^{(-t)}(u_{pt} | u_{1t}, u_{2t}, \dots, u_{(p-1)t}, Z_t) \right]^2 \right\}$$

and the copula candidate that yields the minimum CVPE is selected.

4. DATA

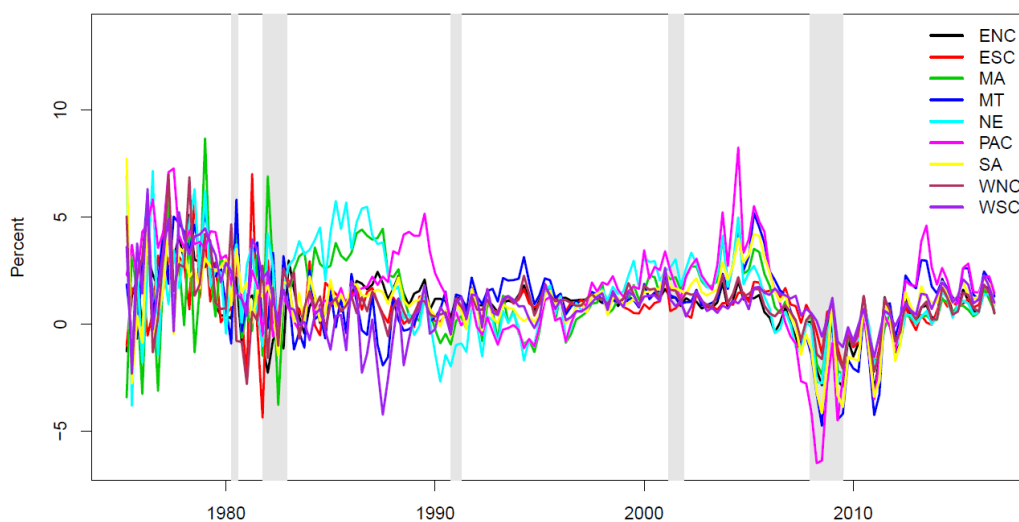
We collect quarterly Housing Prices Index (HPI) in nine census divisions (New England (NE), Middle Atlantic (MA), East North Central (ENC), West North Central (WNC), South Atlantic (SA), East South Central (ESC), West South Central (WSC), Mountain (MT) and Pacific (PA)) from U.S. Federal Housing Finance Agency (FHFA). The data span from 1975:Q1 to 2016:Q4, a period witnesses five recessions.³ Figure 2 demonstrates the path of each division's percentage changes

² See p.55 in Abegaz et al.(2012).

³ According to NBER, the five recession periods are Jan-Jul 1980, Jul 1981-Nov 1982, Jul 1990-Mar 1991, Mar

from the preceding quarter in HPI and the five economic recessions (indicated by the five shaded areas). The greatest volatility in home price happened in the late 1970s and early 1980s. Since then, home prices in most areas became less volatile and appeared to follow an upward trend. The sharp turnaround happened in 2006, the eve of the subprime mortgage crisis caused substantial drop in home price for almost the whole country. Of the nine census divisions, the Pacific division (including California, Oregon, Washington, Alaska and Hawaii) displays the greatest fluctuation in home price after 2000. This finding is consistent with Shiller (2007), who points out that the recent nationwide home price boom that started in 1998 was triggered by the sharp home price increase in west coastal cities.

FIG. 2. Quarterly Percentage Change of HPI in 9 Census Divisions



Note: Shaded areas indicate U.S. economic recessions.

Table 2 documents the descriptive statistics of the quarterly HPI growth rate in each division. The average growth rates are positive in all areas. The estimated first-order autocorrelation coefficients $\hat{\rho}_1$ are positive and statistically different from zero. However, preliminary examinations suggested that autoregressive residuals and autoregressive conditional heteroscedasticity exist in all areas, while the Ljung-Box test indicates that the series of growth rate are serially correlated up to the fourth-order. To avoid spurious dependence, we follow Chen and Fan (2006) and use AR(p)-GARCH(1,1) models for filtering, while the order of AR, p , is selected based on Bayesian Information Criterion (BIC). For example, we fit a series of growth rate to an AR(1)-GARCH(1,1)

process specified as $y_{it} = \delta_i + \gamma_{i1}y_{i,t-1} + \varepsilon_{it}$ where y_{it} denotes the growth of home price at time t

for division i and $\varepsilon_{it} = \sigma_{it} \times e_{it}$ with e_{it} following a Normal distribution with mean zero and

conditional variance defined as $\sigma_{it}^2 = \omega_i + \alpha_{i1}\varepsilon_{i,t-1}^2 + \beta_{i1}\sigma_{i,t-1}^2$.

TABLE 2.

Descriptive Statistics

Census Division	μ	σ	Skewness	Kurtosis	$\hat{\rho}_1$ (p -value)	$LB(4)$ (p -value)
New England	0.0133	0.0207	-0.3292	3.0127	0.540 (0.00)	196.91 (0.00)
Middle Atlantic	0.0116	0.0188	0.4089	4.3577	0.358 (0.00)	100.69 (0.00)
East North Central	0.0093	0.0124	-0.4723	5.1899	0.599 (0.00)	188.58 (0.00)
West North Central	0.0022	0.0138	1.0254	7.6951	0.252 (0.00)	87.61 (0.00)
South Atlantic	0.0105	0.0158	-0.2513	5.9615	0.461 (0.00)	147.89 (0.00)
East South Central	0.0099	0.0143	0.5985	9.5373	0.085 (0.07)	42.68 (0.00)
West South Central	0.0098	0.0137	0.3657	5.3837	0.457 (0.00)	163.31 (0.00)
Mountain	0.0116	0.0185	-0.3755	4.3248	0.568 (0.00)	195.35 (0.00)
Pacific	0.0155	0.0225	-0.4068	4.6189	0.834 (0.00)	371.25 (0.00)

Note: This table documents the summary of quarterly growth rate of for the 9 U.S. census divisions from 1975:Q1 to 2016:Q4. μ is the mean, σ is the standard deviation. $\hat{\rho}_1$ is the first-order autocorrelation with p -value in parenthesis. $LB(4)$ is the Ljung-Box test to the fourth order autocorrelation with p -value in parenthesis.

TABLE 3.

Results of AR(p)-GARCH(1,1) filtering

Census Division	AR(p) part (up to order 3)			GARCH(1,1) part		
	γ (p -value)	ω (p -value)	α_1 (p -value)	β_1 (p -value)		
New England	0.489, 0.089, 0.297 (0.00) (0.30) (0.00)	0.000 (0.22)	0.112 (0.02)	0.848 (0.00)		
Middle Atlantic	0.483, -0.082, 0.467 (0.00) (0.27) (0.00)	0.000 (0.07)	0.285 (0.00)	0.668 (0.00)		
East North Central	0.468, -0.166, 0.346 (0.00) (0.08) (0.00)	0.000 (0.18)	0.255 (0.01)	0.760 (0.00)		
West North Central	0.351, 0.098 (0.00) (0.24)	0.000 (0.04)	0.266 (0.00)	0.714 (0.00)		
South Atlantic	0.401, 0.112, 0.232 (0.00) (0.11) (0.00)	0.000 (0.05)	0.272 (0.00)	0.684 (0.00)		
East South Central	0.298, 0.036, 0.331 (0.00) (0.64) (0.00)	0.000 (0.00)	0.343 (0.00)	0.631 (0.00)		
West South Central	0.452, -0.070, 0.371 (0.00) (0.29) (0.00)	0.000 (0.05)	0.488 (0.00)	0.509 (0.00)		
Mountain	0.472, -0.036, 0.350 (0.00) (0.69) (0.00)	0.000 (0.16)	0.269 (0.00)	0.720 (0.00)		
Pacific	0.732, -0.087, 0.255 (0.00) (0.40) (0.00)	0.000 (0.19)	0.257 (0.02)	0.603 (0.00)		

The results of AR(p)-GARCH(1,1) for each division are presented in Table 3. For some divisions, an AR(2) process is sufficient to capture the autocorrelation of the quarterly growth while others need an AR(3) process. Table 3 summarizes the coefficients of AR(p)-GARCH(1,1) filtering and most of the results are statistically significant. After estimation, a new series of \tilde{y}_i ($i = 1, 2, \dots, 9$) are calculated as $\tilde{y}_i = \frac{\hat{\varepsilon}_i}{\sqrt{\hat{\omega}_i + \hat{\alpha}_i \hat{\varepsilon}_{i,t-1}^2 + \hat{\beta}_i \hat{\sigma}_{i,t-1}^2}}$. Thus, we create a new “filtered” series ($\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_9$) which excludes the potential autoregressive and GARCH effect. In the next step, we will substitute the filtered HPI growth rates into copulas to estimate how the dependence structure, measured by the copula parameter, evolves with respect to some macroeconomic factors such as GDP growth and unemployment rate.

5. TIME-VARYING DEPENDENCE STRUCTURE

Before examining how the dependence structure across different areas adjusts to some fundamental economic factors, we first investigate whether the dependence itself is time-varying over the 40 years.

One widely used parametric method to estimate such a dynamic correlation in multivariate models is the dynamic conditional correlation (DCC) respectively proposed by Tse and Tsui (2002) and Engle (2002). They assume that, for k assets, the conditional correlation matrix ρ_t follows the model $\rho_t = (1 - \gamma_1 - \gamma_2)\rho + \gamma_1\rho_{t-1} + \gamma_2\Psi_{t-1}$ where γ_1 and γ_2 are scalar parameters, ρ is a $k \times k$ positive-definite matrix with unit diagonal elements, and Ψ_{t-1} is the $k \times k$ correlation matrix. Engle (2002) proposes that $\rho_t = D_t Q_t D_t$ where $Q_t = (q_{ij,t})_{k \times k}$ is a positive definite matrix that satisfies $Q_t = (1 - \gamma_1 - \gamma_2)\bar{Q} + \gamma_1\eta_{t-1}\eta'_{t-1} + \gamma_2 Q_{t-1}$ and $D_t = \text{diag}\left(1/\sqrt{q_{11,t}}, \dots, 1/\sqrt{q_{kk,t}}\right)$. Here, η_t is the standardized innovation vector, \bar{Q} is the unconditional covariance matrix of η_t , and γ_1 and γ_2 are nonnegative scalar parameters. D_t matrix is a normalizing matrix to make ρ_t is a correlation matrix. Engle (2002) recommends a two-step method to estimate this model. Hafner and Reznikova (2010) propose a semiparametric method to estimate the dynamic copula which allows the copula parameter to vary over time. Their semiparametric method includes the estimation of the marginals at the first stage and the estimation of the copula parameter after replacing unknown marginals with

the estimated ones at the second stage. In our case, we impose log transformation on

$$c(F_1(x_{1t}), F_2(x_{2t}), \dots, F_9(x_{9t})) = \frac{\partial^9 C}{\partial F_1 \partial F_2 \dots \partial F_9} \cdot f_1 \times f_2 \times \dots \times f_9$$

which is the PDF version of $F(x_{1t}, x_{2t}, \dots, x_{9t}) = C(F_1(x_{1t}), F_2(x_{2t}), \dots, F_9(x_{9t}); \theta)$. Then, we obtain the log likelihood for a sample with T observations:

$$\begin{aligned} L(\theta) &= \sum_{t=1}^T L_t(\theta) \\ &= \sum_{t=1}^T \left\{ \log \frac{\partial^9 C_t(F_1(x_{1t}), F_2(x_{2t}), \dots, F_9(x_{9t}))}{\partial F_1 \partial F_2 \dots \partial F_9} \right\} + \sum_{t=1}^T \{ \log f_1(x_{1t}) + \log f_2(x_{2t}) \dots + \log f_9(x_{9t}) \} \\ &= L_C(\theta) + L_V \end{aligned}$$

Since the second term, L_V , does not contain θ , we concentrate on the first term $L_C(\theta)$ to estimate θ .

In this analysis, for simplicity, we estimate all the marginals via rescaled empirical CDFs at the first stage:

$$\hat{F}_i(x_i) = \frac{1}{T+1} \sum_{t=1}^T I(x_{it} \leq x_i), i = 1, 2, \dots, 9$$

where $I(\bullet)$ is an indicator function. We replace the unknown marginals with the estimated empirical CDFs. Then, at the second stage, define the local likelihood function

$$\tilde{L}(\theta; h, \tau) = \sum_{t=1}^T l(\hat{F}_1(x_{1t}), \hat{F}_2(x_{2t}), \dots, \hat{F}_9(x_{9t}); \theta) \cdot K_h(t/T - \tau)$$

where $\tau \in [0, 1]$, $l(\hat{F}_1(x_{1t}), \hat{F}_2(x_{2t}), \dots, \hat{F}_9(x_{9t}); \theta) = \log \frac{C_\theta(\hat{F}_1(x_{1t}), \hat{F}_2(x_{2t}), \dots, \hat{F}_9(x_{9t}))}{\partial \hat{F}_1 \partial \hat{F}_2 \dots \partial \hat{F}_9}$,

$K_h(\cdot) = K(\cdot/h)/h$ is the kernel function with bandwidth h selected through an Extended Residual Square Criterion (ERSC) suggested by Fan et al. (1998). Then, the local likelihood estimator of the function $\theta(\tau)$ could be obtained through $\hat{\theta}(\tau) = \arg \max_{\theta} \tilde{L}(\theta; h, \tau)$.

We apply this method to Gaussian, Clayton, Gumbel and Frank copula, respectively. Since different copulas' parameters are not directly comparable, we convert the time-varying copula parameter $\hat{\theta}_t$ into Kendall's τ_t and plot them (solid line) in Figure 3. All the four copulas display similar patterns of Kendall's τ over the 40 years. For model selection, following Hafner and Reznikova (2010), we choose Gaussian copula because it exhibits the lowest BIC among the four

candidates for the case of time-invariant copula parameters as shown in Table 4. The plot displays how the dependence structure across the 9 housing markets evolved over 1975-2016. Before the mid-1980s, regional home prices appeared to be mutually irrelevant because of the low association. This is consistent with our observation in Figure 2 which exhibits some volatility before the mid-1980s. Since then, the dependence started an upward trend, increasing remarkably from 0.1 to 0.7 in 2000. After a temporary adjustment during early 2000s, following the collapse of the housing market in 2006, regional home prices resumed strong association as was in the 1990s. For comparing purposes, we additionally calculate the Kendall's τ obtained from each time-invariant copula and draw it in the plot (blue dot-dashed line), respectively. The time-invariant Kendall's τ s appear to underestimate the strengthened dependence after 1990s but overestimate the low degree of association before 1980. Such a difference should be expected because the time-invariant Kendall's τ is a balance of strong and weak dependence in regional home prices over the whole sample period. According to Figure 3, the correlation of regional housing prices indeed behaved differently over the 40 years and the degree of association was strengthened after the recent recession.

FIG. 3. Transform to Kendall's τ of the estimated time-varying copula dependence parameter $\theta(\tau)$ for the nine census divisions.

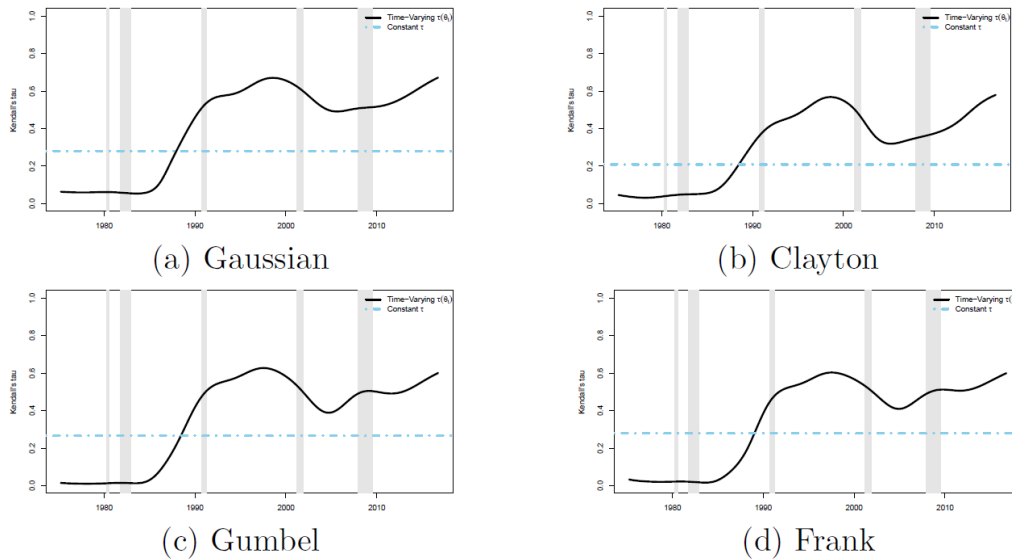


TABLE 4.

Fit of Gaussian, Clayton, Gumbel and Frank for the 9 Census Divisions with the estimated time-invariant dependence parameter θ and Kendall's τ .

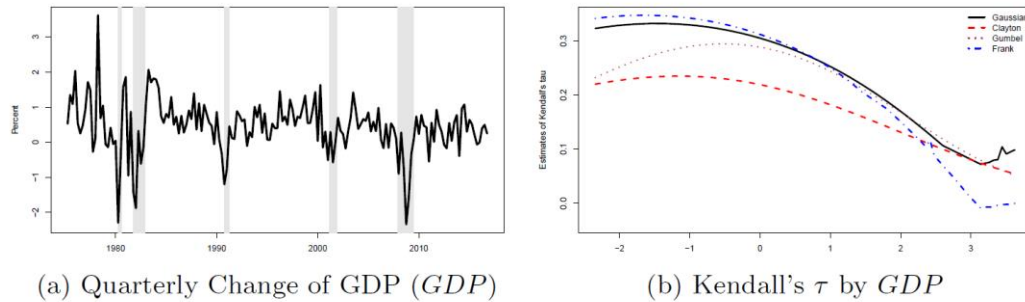
Copula	BIC	τ (<i>S.E.</i>)	θ (<i>S.E.</i>)
Gaussian	-230.882	0.28 (0.00)	0.426 (0.021)
Clayton	-221.282	0.21 (0.00)	0.528 (0.036)
Gumbel	-230.800	0.38 (0.00)	1.626 (0.073)
Frank	-153.162	0.28 (0.00)	2.687 (0.173)

As we discussed earlier, compared with DCC and the dynamic factor model by Kallberg et al. (2014), the main advantage of copula is that it is able to detect the nonlinear dependence structure among multiple assets simultaneously and identify their comovement during some extreme scenarios. For comparing purposes, we also apply DCC and calculate the pairwise dynamic correlations between several regions. Figure A1 in the supplementary appendix shows that even though DCC finds fluctuating correlations between regional housing markets, it does not effectively capture the sharp increase in dependence among these regions during the most recent housing price boom. Kallberg et al. (2014)'s dynamic factor model indeed finds the strengthened correlation among the 14 large U.S. cities since 2000, but fails to identify the remarkable comovement during the 1990s. This comparison provides further evidence on copula's ability to capture the tail dependence among multiple assets. In the next section, we will use the conditional copula to investigate how the correlation adjusts to some fundamental economic factors.

6. ESTIMATION RESULTS

We collect 6 national economic factors from the Federal Reserve Economic Data (FRED): quarterly growth of per capita real GDP (GDP), residential investment as a percent of GDP (INV), mortgage debt service payments as a percent of disposable personal income (DEBT), civilian unemployment rate (UNE), quarterly growth of real disposable personal income (INC), and real federal funds rate (RINT). All these factors can, to some extent, mirror the macroeconomic situation and are tracked closely by investors and policy makers.

FIG. 4. Quarterly percentage change of GDP (Q2:1975-Q4:2016) and four copulas' paths of Kendall's τ along with the quarterly percentage change of GDP. The percentage change of GDP is calculated based on the real GDP inflated by 2009 dollar. The shaded areas represent the date of U.S. recessions computed by NBER.



Revenue from housing market is an important source of the national account and housing's combined contribution to GDP is about 15-18% on average (Bureau of Economic Analysis). Figure 4a displays the quarterly growth of per capita real GDP from 1975 to 2016 and it is evident that per capita GDP declined substantially during the five recessions. We plot the relationship between the interdependence across the 9 regional housing markets and the growth of GDP in Figure 4b. According to Figure 4b, all four types of copula exhibit similar paths of Kendall's τ and the magnitude of dependence is relatively larger when the economy becomes contracted. In other words, regional home prices tend to crash simultaneously when a recession is looming. But such a lower-tail dependence is weakened when GDP resumes positive growth. Kendall's τ decreases from 0.3 to about 0.1. In general, the magnitude of dependence is low when we use GDP as the covariate. One possible explanation is that, as indicated in Figure 2, the comovement across housing markets in different areas are not substantial or only appeared in some of those areas during the normal periods due to some regional-specific idiosyncratic factors which are not controlled by conditional copulas.

Housing's contribution to GDP is mainly through two channels. The first channel is the consumption spending on housing services, which increased from 3,992 billion dollars in 1980 to 11,572 billion dollars in 2016 (Bureau of Economic Analysis). The second channel is the residential investment, which includes construction of new single-family and multifamily structures, residential remodeling, production of manufactured homes, and brokers fees. From 1980 to 2005, the residential investment increased from 333 billion dollars to 873 billion dollars, but then sharply dropped to 382 billion dollars in 2010. Shiller (2007) identifies that residential investment is highly correlated with the business cycle and, as documented in Figure 5a, residential investment as a percent of GDP has notably decreased during four marked recessions, except for the one in 2001.

FIG. 5. Residential investment as percent of GDP (Q2:1975-Q4:2016) and four copulas' paths of Kendall's τ along with residential investment as percent of GDP. The shaded areas represent the date of U.S. recessions computed by NBER.

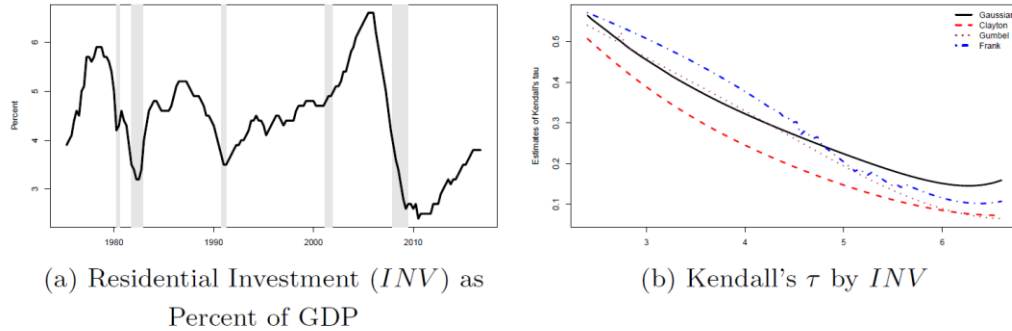
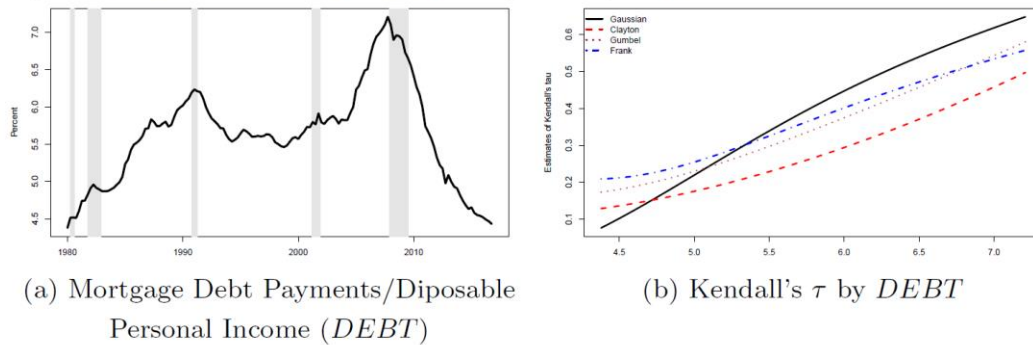


Figure 5b shows that the degree of dependence is larger when the portion of residential investment in GDP is low. In other words, it implies that the strongest interdependence is expected when the residential investment level remarkably drops, which usually occurs in times of crisis. This finding is consistent with the result conditional on GDP growth, and compared with Figure 4b, the degree of dependence becomes much larger when using the ratio of residential investment in GDP as the covariate.

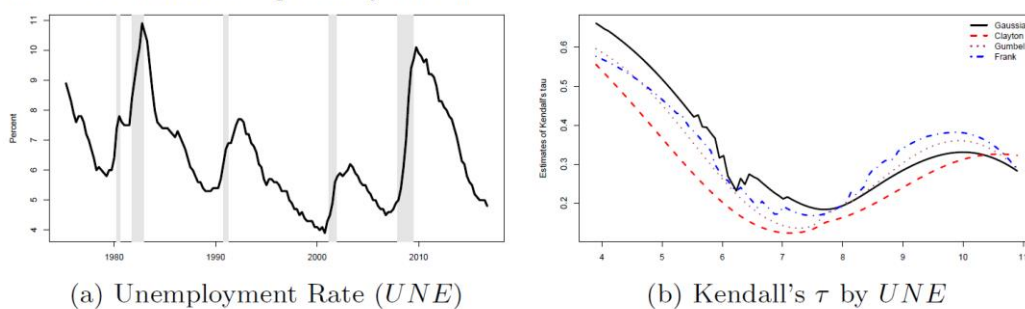
FIG. 6. Mortgage debt service payments as a percent of disposable personal income (*DEBT*) (Q1:1980-Q4:2016) and four copulas' paths of Kendall's τ along with mortgage debt service payments as a percent of disposable personal income. The shaded areas represent the date of U.S. recessions computed by NBER.



We then examine how the dependence structure adjusts to the mortgage debt service payments as a percent of disposable personal income. Since most housing transactions are debt financed, this ratio can partially measure the individual average debt payment ability. As displayed in Figure 6a, the percentage of mortgage payment is prominently high during all five recessions, and then sharply dropped in the midst of recessions. For example, in the eve of the latest recession in 2007-2008, the ratio of mortgage debt payments in disposable personal income reached its historical high at about 7.1%. However, following the burst of housing bubble, banks have tightened their lending criteria and adjustable rate mortgages become less common. At the end of 2016, the ratio of mortgage debt

service payments to disposable income decreased to only 4.5%. Landier et al. (2017) provide further evidence about how banking integration propagate regional home price movement to other states: When banks face idiosyncratic shocks and have branches in multiple states, their lending activity induces home price comovement. Such an integration among banks could, to some extent, explain why mortgage debt level is useful to monitor home price dependence, because the banking integration since 1980s makes mortgage debt becomes more convenient and attainable. In Figure 6b, all four copulas give similar patterns of Kendall's τ along with the ratio of mortgage debt payments. The dependence is weak when the portion of mortgage payments is low, but becomes strengthened as the ratio increases. During the so called "irrational exuberance" (Shiller, 2007) and when consumers express mass desires to buy homes, mortgage debts will build up rapidly and home prices in different regions will increase simultaneously, leading to the stronger interdependence.

FIG. 7. Quarterly unemployment rate (Q2:1975-Q4:2016) and four copulas' paths of Kendall's τ along with the unemployment rate. The shaded areas represent the date of U.S. recessions computed by NBER.



High unemployment rate is another feature of economic recessions. Figure 7b indicates that, during the "normal" time or when the unemployment rate is moderate, the association among regional home prices seems to be weak. When job cuts become pervasive, financial pressure caused by lay-off will lower the demand for new houses so that home prices tend to move down. On the other hand, low unemployment rate will spur demands for new houses, and such a high demand will lead to an increase in price, as evidenced by 1990s. Under this circumstances, the association in home prices will also become stronger. Following this argument, we find similar U-shape Kendall's τ curves in Figure 8, which displays how the dependence changes along with the quarterly growth of real personal income. It implies that both substantial increase and decrease in personal income will lead to remarkable synchronization in home prices.

FIG. 8. Quarterly percentage change of real per capita income (Q2:1975-Q4:2016) and four copulas' paths of Kendall's τ along with the quarterly percentage change of real per capita income. The shaded areas represent the date of U.S. recessions computed by NBER.

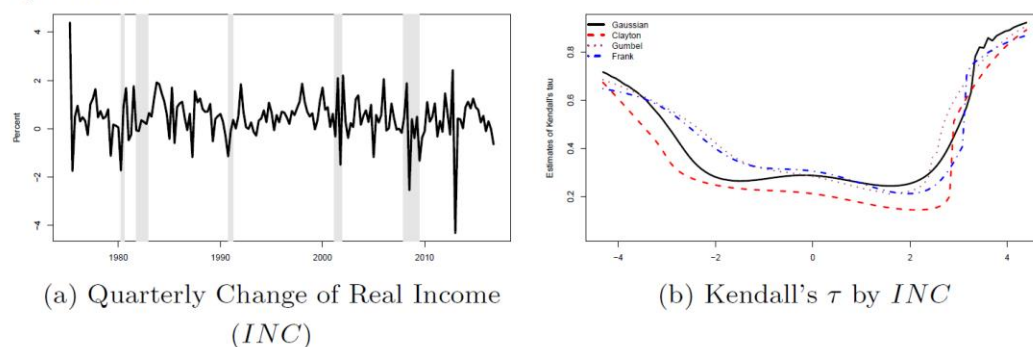
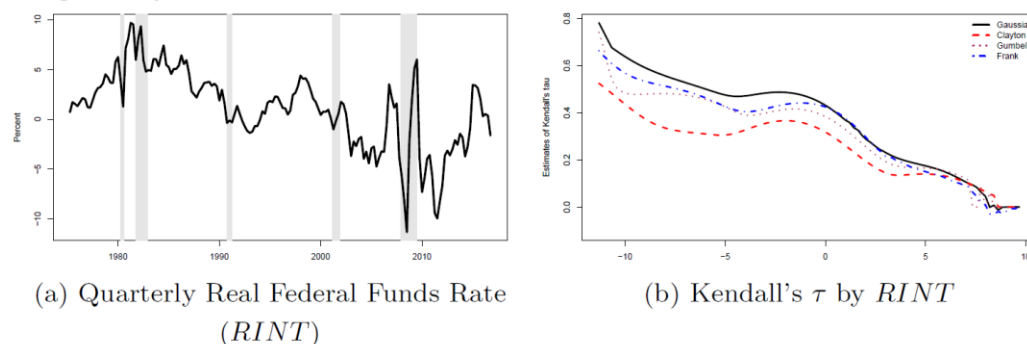


FIG. 9. Quarterly real federal funds rate (Q2:1975-Q4:2016) and four copulas' paths of Kendall's τ along with the quarterly real federal funds rate. The quarterly real federal funds rate is computed by subtracting the rate of increases of the CPI (all items less shelter) for the past 12 months. The shaded areas represent the date of U.S. recessions computed by NBER.



Interest rate is another factor that affects consumers' desire to buy houses. During a booming period, the Fed tends to increase the federal funds rate to cool down the over-heated economy and speculative investment. High interest rate will increase consumers' cost and thus dampen their demands for new homes, leading to a reduction in housing prices. On the contrary, in times of crisis, to stabilize the economy, the Fed often chooses to decrease the interest rate. Figure 9a shows the real federal funds rate computed by subtracting the rate of increases of the CPI (all items less shelter) for the past 12 months decreased sharply in all the five recession since 1975. Correspondingly, demands for houses will be spurred by the low interest rate and the loose monetary policy, leading to a new wave of increase in home price. This hypothesis is confirmed by Figure 9b. It shows that the degree of interdependence across the nine divisions is rather low when the real interest rate is high. When the Fed gradually decreases the interest rate, the interdependence becomes strengthened and its degree will be increased to about 0.6 - 0.8.

In summary, adopting the semiparametric estimation to the conditional copula, we find that the dependence across regional housing markets indeed adjusts to different levels of the fundamental economic factors. Regional housing markets tend to crash simultaneously in times of crisis due to the dampened demands for new houses, which is in line with Rodriguez (2007), Zimmer (2012) and Kallberg et al. (2014). Rapid expansion in personal mortgage debt is another important reason for strengthened synchronization among regional housing markets. On the other hand, we find that home price is not monotonically correlated to unemployment rate and personal income. Such an interesting finding implies that correlation in home price is partially determined by the demand side: a booming economy and an active labor market generate more demands for houses and spur home price, while a dim economy and lowered income dampen housing demands and lead to a downward comovement. For policy makers, our findings provide further evidence that demand and desire of purchasing houses exhibit large impact on home price. Policies aiming to encourage personal mortgage and labor participation will not only spur home price, but also promote synchronization among regional housing markets.

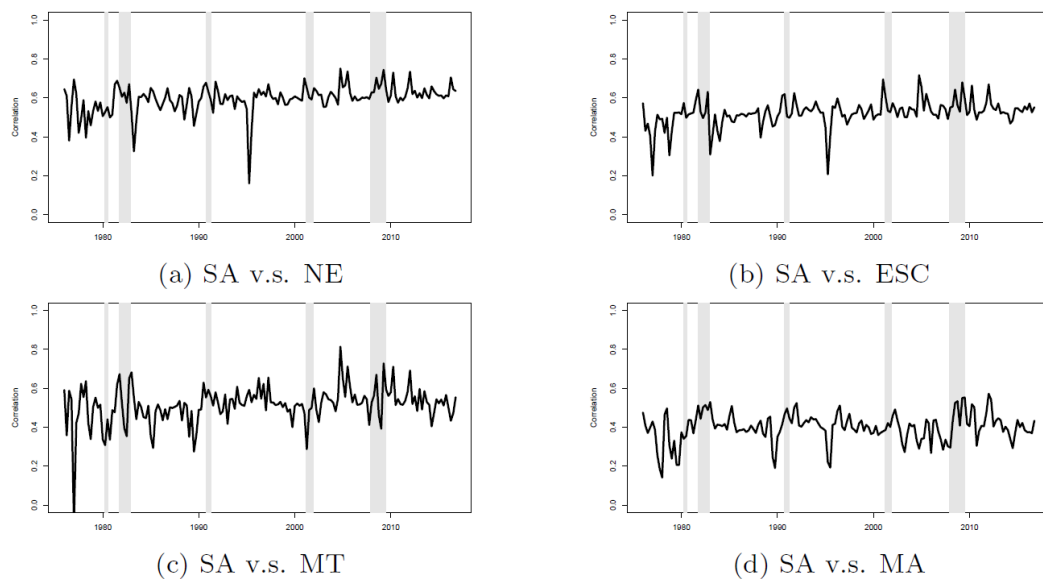
There are a number of caveats to these findings. One concern is that there exists many other regional and even metro-specific idiosyncratic factors that drive the comovement within that area and those factors may explain a large portion of the volatility (Fu, 2007). Our analysis only considers how national factors, such as interest rate, affect the dependence structure. However, considering that the recent boom in housing price is a nationwide phenomenon, we believe that the effect of state- or regional- specific factors is relatively small. The second limitation is that, even though the comovement adjusts to the six national economic factors, we cannot identify which are the determinants of dependence, because many factors are closely correlated to each other. One way to select the useful covariates is to extend the current univariate x in $\theta(x)$ to a p -dimension, say $\theta(\mathbf{X}_t)$, where $\mathbf{X}_t = \{x_{1t}, x_{2t}, \dots, x_{pt}\}'$, and then choose those exhibit strong explaining power to the variation of the copula parameter θ under certain selection criterion. This will extend the current conditional copula to a single index copula, which deserves another research in the future and is beyond the scope of this study.

7. CONCLUSION

We adopt a semiparametric method to study how the dependence across housing markets in nine U.S. census divisions evolves along with several economic indicators. We first prove that the U.S. residential real estate market have become more integrated since the mid-1980s and the conventional time-invariant copula fails to capture the substantially strengthened association during the economic expansion periods. Then we identify the relationship between the dependence and six fundamental economic factors and conclude that the association among regional housing markets is affected by the macro economic situation and the monetary policy. Even though conditional copula is unable to identify the causal relationship between the dependence structure and those economic factors, our findings will help investors and home buyers to analyze and evaluate the systematic risk in the nationwide housing market. A more thorough analysis to identify which factors determine the dependence structure requires a more generalized conditional copula model, which will inspire more comprehensive researches in the future.

APPENDIX

FIG. 1. Time-varying correlations estimated by DCC. SA = South Atlantic, NE = New England, ESC = East South Central, MA = Middle Atlantic, MT = Mountain.



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